Exercise 219

The cost to remove a toxin from a lake is modeled by the function $C(p) = 75p/(85 - p)$, where C is the cost (in thousands of dollars) and p is the amount of toxin in a small lake (measured in parts per billion [ppb]). This model is valid only when the amount of toxin is less than 85 ppb.

a. Find the cost to remove 25 ppb, 40 ppb, and 50 ppb of the toxin from the lake.

b. Find the inverse function. c. Use part b. to determine how much of the toxin is removed for $50,000.

Solution

Part (a)

Plug in $p = 25$, $p = 40$, and $p = 50$ in the given function for $C(p)$.

$$p = 25 \qquad \Rightarrow \qquad C(25) = \frac{75(25)}{85 - (25)} = 31.25 = \$31{,}250$$

$$p = 40 \qquad \Rightarrow \qquad C(40) = \frac{75(40)}{85 - (40)} \approx 66.667 = \$66{,}667$$

$$p = 50 \qquad \Rightarrow \qquad C(50) = \frac{75(50)}{85 - (50)} \approx 107.143 = \$107{,}143$$

Part (b)

Solve the given function,

$$C(p) = \frac{75p}{85 - p},$$

for p.

$$C = \frac{75p}{85 - p}$$

$$C(85 - p) = 75p$$

$$85C - pC = 75p$$

$$85C = 75p + pC$$

$$85C = (75 + C)p$$

$$\frac{85C}{75 + C} = p$$

Therefore, the function that converts from cost to parts per billion is

$$C^{-1}(C) = \frac{85C}{75 + C}.$$

Part (c)

Plug in $C = 50$ to the inverse function to find how many parts per billion can be removed for \$50,000.

$$C = 50 \qquad \Rightarrow \qquad C^{-1}(50) = \frac{85(50)}{75 + (50)} = 34 \text{ ppb}$$

Therefore, 34 parts per billion can be removed for \$50,000.